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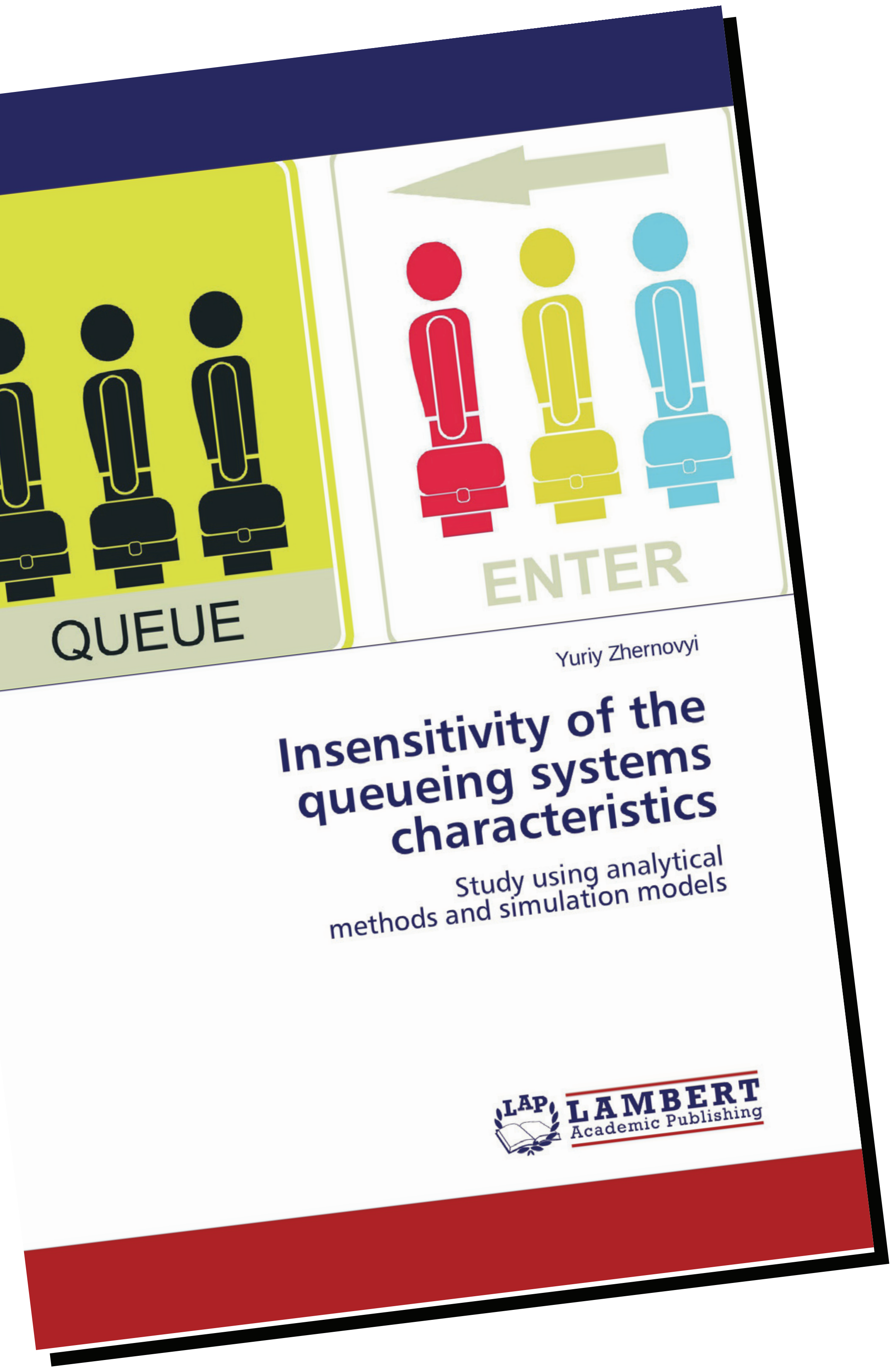

Insensitivity of the queueing systems characteristics

Study using analytical methods and simulation models

Yuriy Zhernovyi

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This monograph is devoted to the study of the insensitivity property of the stationary characteristics of queueing systems using analytical methods and simulation models. For systems with rejections, consisting of heterogeneous channels, we examine the insensitivity of the stationary characteristics with respect to the form of the service-time distributions. We consider three ways of distribution of customers by channels of the queueing system: equiprobable distribution by free channels, equiprobable distribution by all channels, and ordered distribution. For systems with waiting we define the stationary characteristics that are insensitive with respect to the form of the distribution of service time and interarrival time of customers. For certain queueing systems with threshold switching of functioning modes we prove the insensitivity of the stationary characteristics with respect to the form of the service-time distribution in the state of overload mode. The book is intended for researchers and students engaged in the study of queueing systems.

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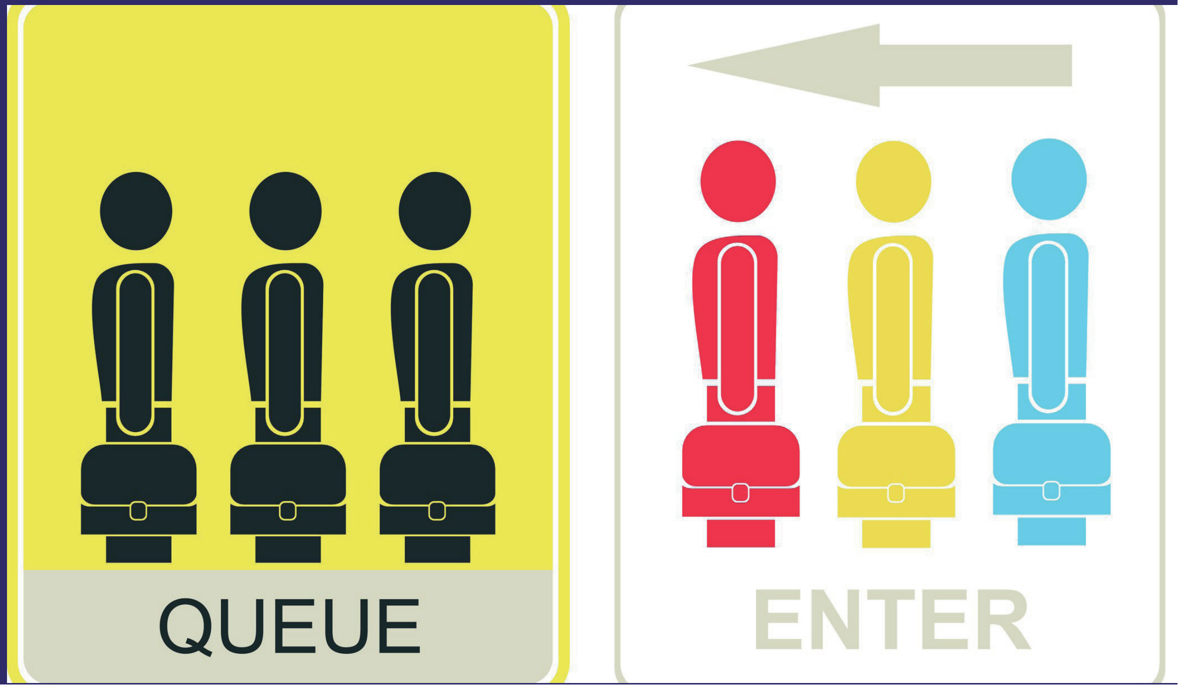

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INTRODUCTION

Let us consider the queueing system with rejections of the M/M/n/0 type. It is the system with a stationary Poisson input flow of rate λ and exponential distributed service times with parameter μ. The stationary distribution (p_k) of the number of busy channels is a function of λ and μ. Let $\tau = 1/\mu$, then $(p_k) = f(\lambda, \tau)$. The question arises: is this formula valid for an arbitrary service-time distribution $G(x)$ for a fixed average τ? This property has been called *invariance* or *insensitivity* (the distribution (p_k) with respect to the service-time distribution with the same mean τ).

The researchers studied the problem of insensitivity since the middle of the twentieth century. B. A. Sevastyanov [18] has proved the insensitivity property for the M/G/n/0 system by means of integro-differential equations method. The other proofs of the independence of the distribution (p_k) of multi-channel system with rejections on the type of distribution $G(x)$ for a fixed average service time [7, 11, 15] appeared later.

I. N. Kovalenko found necessary and sufficient conditions of the insensitivity of the M/G/∞ system, that receives the customers of different types [12]. B. T. Guseinov [8] and D. König [10] considered the generalization of Kovalenko's theorem.

This monograph consists of three chapters. In the first chapter we have proved the insensitivity of the stationary characteristics with respect to the form of the service-time distributions for the two M/G/n/0 systems with heterogeneous channels and different ways of distribution of customers by the channels (the equiprobable customer distribution by all channels and the equiprobable customer distribution by free channels). We have established that for systems with rejections that are near to these two types of systems, the insensitivity property is not saved.

The Little's formula, connecting the mean waiting time $\mathrm{E}(W)$ with the mean queue length $\mathrm{E}(Q)$,

$$\mathrm{E}(W) = \frac{\mathrm{E}(Q)}{\lambda_{\mathrm{sv}}},$$

is valid in the steady state for each queueing system with stationary flows of customers and services. Here, $\lambda_{\mathrm{sv}} = \lambda \mathrm{P}_{\mathrm{sv}}$ is the flow rate of serviced customers, P_{sv} is the stationary probability of service for customer, and λ is the intensity of the input flow. For the system without losses of customers we have $\mathrm{P}_{\mathrm{sv}} = 1$ and $\lambda_{\mathrm{sv}} = \lambda$. It follows from the Little's formula that the steady-state characteristic $\omega_{\text{sv}} = \mathrm{E}(Q)/(\mathrm{P}_{\text{sv}}\mathrm{E}(W))$ is insensitive with respect to the form of distribution functions of service time and interarrival time. The insensitivity of some other stationary characteristics can be proved using the balance equation for the average number of arrivals and serviced customers per unit of time, which is valid in the steady state.

However, for complicated queueing systems (for example, for the systems with general distributions and batch arrivals) is usually not possible to prove the existence of a limiting steady state by analytical methods. In this case, we can try to check the alleged condition of the stationary distribution existence and the insensitivity of the stationary characteristics using simulation models. This approach is proposed in the second chapter of this paper. For the construction of simulation models, we use the GPSS World simulation system [2, 4, 13, 19, 25]. Simulation models are used by us to verify and illustrate the results obtained by analytical methods.

Obtaining of recurrence relations or explicit formulas is the most common way of establishing the insensitivity of the stationary characteristics of queueing systems. In the third chapter of the monograph we give such relations obtained in recent studies [20-22] for queueing systems with threshold functioning strategies. Some of the explicit formulas obtained by us, are published for the first time.

If the insensitivity of characteristics is proved for some system, the question arises about the verification of insensitivity properties of these characteristics for near to that considered here, but a more general system (for example, with the input flow of a more general type). In some cases, manage to prove sensitivity of the characteristics of such system, using the Erlangian distribution and the method of phases. This method is based on the fact that a random variable, distributed according to the Erlang law of k-th order, is the sum of k independent exponentially distributed random variables. We use the method of phases to study the insensitivity property in the first and third chapters.

Let us consider the basic assumptions and notation used in this paper.

The interarrival times T_{ar} and the service times T_{sv} assumed to be independent identically distributed random variables with finite mean values $\mathrm{E}(T_{\text{ar}}) = 1/\lambda < \infty$ and $\mathrm{E}(T_{\text{sv}}) = \tau < \infty$.

We introduce the notation: $F(x)$ is the probability distribution function of the random variable T_{ar}, $G(x)$ is the probability distribution function of the random variable

T_{sv}, X is the number of customers in the batch, a_k ($1 \le k < \infty$) is probability of the event $\{X=k\}$, moreover $\mathrm{E}(X) = \sum_{k=1}^{\infty} k a_k < \infty$, $\sum_{k=1}^{\infty} a_k = 1$.

To refer of the systems with batch arrivals we use the superscript X. For example, $\mathrm{G}^{\mathrm{X}}/\mathrm{G}/1$ is the G/G/1 system, in which customers arrive in batches and the batch size is distributed according to the random variable X.

We consider the following stationary characteristics of the queueing systems: N_{c} is the number of customers in the system, p_k is the probability of the event $\{N_{\mathrm{c}}=k\}$, $\mathrm{E}(Q)$ is the mean queue length, $\mathrm{E}(W)$ is the mean waiting time in the queue, P_{sv} is the probability of service for arrived customer, $\mathrm{P}_{\mathrm{rej}}$ is the probability of rejection, $(\mathrm{P}_{\mathrm{sv}} = 1 - \mathrm{P}_{\mathrm{rej}})$, $\rho = \lambda \tau \mathrm{E}(X)$ is the load factor, $\mathrm{E}(n_{\mathrm{bs}}) = \overline{U}$ is the average number of busy channels, $\mathrm{E}(T_0)$ is the mean length of the idle period (period of time when there is no customers in the system), $\mathrm{E}(T_{\mathrm{bs}})$ is the mean length of the busy period.

The results of studies, presented in this monograph, were published in the author's papers [22, 24]. In the third chapter, we use the recurrence relations for the stationary characteristics of the queueing systems with threshold functioning strategies, obtained in [20, 21]. The results, presented in Sections 1.2.3, 1.2.5, 1.3.1-1.3.5, 3.3.4-3.3.7, 3.4.4 and 3.4.5, is published for the first time.

1 SYSTEMS WITH REJECTIONS

Queueing systems with rejections (without waiting) are employed to calculate the number of communications channels (physical or logical links) that is required to ensure the specified quality of service when information is transmitted in modern telecommunications networks [5] and to solve the problems related to the investigations of circuit-switched networks, digital integrated-service networks, and adaptive terminal measuring systems [16].

In this chapter, we examine the insensitivity of the stationary characteristics of systems with rejections, consisting of heterogeneous channels. Will be considered three ways of distribution of customers by channels of the system: equiprobable distribution by free channels, equiprobable distribution by all channels, and ordered distribution.

The system where customers are equiprobably distributed by all channels can be used as the queueing model characterizing the violations in the control of customer distributions over servers. Such a system was discussed in [17] under the assumption of the exponential service-time distribution and homogeneous servers. Multiserver queueing systems with heterogeneous servers arise in various applications, in particular, they are an adequate model of the communication node of data transmission [6].

For the $M/G/n/0$ system with equiprobable distribution of customers by all channels we will use the notation $M_{1/n}/G/n/0$, and let $M_{1/n}/G_1,\ldots,G_n/n/0$ denotes the corresponding system with heterogeneous channels. Under the heterogeneity of channels we mean unequal service-time distribution in different channels of the system. Let $M/G_1,\ldots,G_n/n/0$ denotes the system with heterogeneous channels and ordered distribution of customers by free channels. For the ordered distribution preference is given to the channel with the lowest number, and in the case of homogeneous channels, the $M/G_1,\ldots,G_n/n/0$ system turns into the $M/G/n/0$ system.

We denote by $M_{1/n(\mathrm{free})}/G_1,\ldots,G_n/n/0$ the system with heterogeneous channels, which applies equiprobable distribution of customers by free channels. In the case of homogeneous channels the $M_{1/n(\mathrm{free})}/G/n/0$ and $M/G/n/0$ systems coincide.

1.1 Systems with heterogeneous channels and equiprobable distribution of customers by free channels

1.1.1 Insensitivity of the characteristics of the $\mathbf{M_{1/n(free)}/G_1,\ldots,G_n/n/0}$ system

Let us consider an $\mathrm{M_{1/n(free)}/G_1,\ldots,G_n/n/0}$ queueing system with rejections, which involves n heterogeneous channels (with different service time distributions) and receives the stationary Poisson flow of customers with intensity λ. If all n channels are busy at the instant of arrival, the failure of servicing occurs (i.e., a customer leaves the system without being processed). In the case when k channels are busy, the customer comes to any of the free channels with probability $1/(n-k)$. In the ith channel, a customer service time is a random variable with distribution function $G_i(x)$ and mathematical expectation τ_i.

It is of interest to consider the stationary characteristics

$$p_k = \lim_{t\to\infty} p_k(t), \qquad 0 \le k \le n,$$

where $p_k(t)$ is the probability that customers are served by k channels at instant t.

Let $\nu(t)$ denotes the number of busy channels at instant t. Let us assume that, at a certain instant, $\nu(t)$ is equal k $(1 \le k \le n)$, under the condition of $\nu(t-0) \ne k$, and that numbers from 1 to k be randomly assigned to channels employed at instant t. The assigned numbers remain in force until $\nu(t)$ takes a new value. Let $\xi_j(t)$ denotes the time interval between t (service continues at time t) and the instant when the jth device completes servicing. Then it is possible to consider the random process $\zeta(t) = \{\nu(t); \xi_1(t), \xi_2(t), \ldots, \xi_{\nu(t)}(t)\}$.

Let us introduce the following notation:

$$F_k(t; x_1, x_2, \ldots, x_k) = \mathbf{P}\{\nu(t) = k; \xi_1(t) < x_1, \xi_2(t) < x_2, \ldots, \xi_k(t) < x_k\},$$
$$F_{k(i_1, i_2, \ldots, i_k)}(t; x_1, x_2, \ldots, x_k) =$$
$$= \mathbf{P}\{\nu(t) = k; \xi_{1i_1}(t) < x_1, \xi_{2i_2}(t) < x_2, \ldots, \xi_{ki_k}(t) < x_k\}, \quad 0 \le k \le n.$$

Here, $\xi_{ji_j}(t)$ is the time interval between t and the instant when the jth channel completes the service under the condition that, in this channel, the service time is distributed according to law $G_{i_j}(x)$. Moreover, all possible ordered sets $(i_1, i_2, \ldots, i_k)$ are regarded to have no identical numbers and $1 \le i_j \le n$ for each value of j.

It is obvious that $p_k(t) = F_k(t; \infty, \infty, \ldots, \infty)$, i.e.,

$$p_k = \lim_{t\to\infty} F_k(t; \infty, \infty, \ldots, \infty).$$

Hence, to determine stationary probabilities p_k, functions F_k must be found.

Let us introduce the following notation: $A_n^k = n!/(n-k)!$ is the number of k arrangements from n,

$$F_k(x_1, x_2, \ldots, x_k) = \lim_{t\to\infty} F_k(t; x_1, x_2, \ldots, x_k),$$
$$F_{k(i_1, i_2, \ldots, i_k)}(x_1, x_2, \ldots, x_k) = \lim_{t\to\infty} F_{k(i_1, i_2, \ldots, i_k)}(t; x_1, x_2, \ldots, x_k).$$

Theorem 1.1. *If* $\tau_i < \infty$ *and* $1 \le i \le n$, *then random process* $\zeta(t)$ *has an ergodic stationary distribution and stationary probabilities* p_k *are determined by the formulas*

$$p_k = p_0 \tilde{p}_k, \qquad 1 \le k \le n; \qquad \frac{1}{p_0} = 1 + \sum_{k=1}^{n} \tilde{p}_k;$$
$$\tilde{p}_k = \frac{\lambda^k}{A_n^k} \sum_{\substack{i_1, i_2, \ldots, i_k = 1; \\ i_1 < i_2 < \ldots < i_k}}^{n} \prod_{j=1}^{k} \tau_{i_j}, \qquad 1 \le k \le n. \tag{1.1}$$

Proof. Since $\zeta(t)$ is a piecewise linear process [7, p. 383], the validity of the first part of Theorem 1.1 follows from the ergodic theorem for piecewise linear Markov processes [7, p. 211].

When the behavior of process $\zeta(t)$ is studied in the steady-state mode, it is necessary to consider all cases favorable to event A defined as

$$\nu(t+h) = k; \quad \xi_1(t+h) < x_1, \quad \xi_2(t+h) < x_2, \ldots, \xi_k(t+h) < x_k,$$

where $x_i > 0$ and $1 \le i \le k$. In the case of the stationary Poisson input flow, the probability that more than one customer arrives during time h is $o(h)$. Hence, there is a need to examine the cases favorable to event A in time interval $(t, t+h)$: (i) there was no arrivals of customers, (ii) a single customer was served, and (iii) a single customer arrived. Let the service of more than one customer or the service together with a customer arrival can be terminated. Then, with allowance for the equality

$$F_k(x_1, x_2, \ldots, x_k) = \frac{1}{A_n^k} \sum_{\substack{i_1, i_2, \ldots, i_k = 1; \\ i_1 \ne i_2 \ne \ldots \ne i_k}}^{n} F_{k(i_1, i_2, \ldots, i_k)}(x_1, x_2, \ldots, x_k),$$

and the reasoning reported in [7, pp. 384-385], the estimate can be derived as

$$\mathbf{P}\{\nu(t) = k; x_1 \le \xi_1(t) < x_1 + h, x_2 \le \xi_2(t) < x_2 + h, \ldots, x_k \le \xi_k(t) < x_k + h\} \le$$
$$\le \frac{(\lambda h)^k}{A_n^k} \sum_{\substack{i_1, i_2, \ldots, i_k = 1; \\ i_1 \ne i_2 \ne \ldots \ne i_k}}^{n} \prod_{j=1}^{k} \left(1 - G_{i_j}(x_j)\right). \tag{1.2}$$

Using arguments presented in [7, pp. 383-386] and estimate (1.2) applied to the determination of functions $F_k(x_1,x_2,\ldots,x_k)$, we obtain the following equations fulfilled almost everywhere:

$$\sum_{j=1}^{k}\frac{\partial F_k}{\partial x_j}-\lambda(1-\delta_{kn})F_k+$$

$$+\frac{\lambda}{k(n-k+1)}\sum_{j=1}^{k}\sum_{\substack{i_1,i_2,\ldots,i_k=1;\\ i_1\neq i_2\neq\ldots\neq i_k}}^{n}F_{k-1(i_1,i_2,\ldots,i_{k-1})}(x_1,\ldots,x_{j-1},x_{j+1},\ldots,x_k)G_{i_k}(x_j)= \tag{1.3}$$

$$=\sum_{j=1}^{k}\frac{\partial F_k(x_1,\ldots,x_{j-1},0,x_{j+1},\ldots,x_k)}{\partial x_j}-(1-\delta_{kn})(k+1)\frac{\partial F_{k+1}(x_1,\ldots,x_k,0)}{\partial x_{k+1}},\ \ 1\le k\le n,$$

where δ_{kn} are the Kronecker symbols.

Direct substitution confirms that the system of Eqs. (1.3) has almost everywhere the nonnegative, absolutely continuous solution of the form:

$$F_k(x_1,x_2,\ldots,x_k)=\frac{\lambda^k}{k!A_n^k}F_0\sum_{\substack{i_1,i_2,\ldots,i_k=1;\\ i_1\neq i_2\neq\ldots\neq i_k}}^{n}\prod_{j=1}^{k}\int_0^{x_j}\left(1-G_{i_j}(u)\right)du. \tag{1.4}$$

Since

$$\lim_{\substack{x_j\to\infty,\\ 1\le j\le k}}F_k(x_1,x_2,\ldots,x_k)=\frac{\lambda^k}{A_n^k}F_0,$$

then under the normalization condition

$$F_0\sum_{k=0}^{n}\frac{\lambda^k}{A_n^k}=1$$

formulas (1.4) present the ergodic distribution of process $\zeta(t)$. In particular, when all coordinates tend to infinity, we obtain formulas (1.1). The theorem is proved. □

Remark 1.1. *It follows from* (1.1) *that the stationary characteristics of the* $\mathrm{M_{1/n(free)}/G_1,\ldots,G_n/n/0}$ *system are insensitive with respect to the service-time distributions*.

For the $\mathrm{M_{1/n(free)}/G_1,\ldots,G_n/n/0}$ system, the stationary probability of rejection is defined as

$$\mathrm{P_{rej}}=p_n=p_0\lambda^n\prod_{j=1}^{n}\tau_j. \tag{1.5}$$

If the service-time means of all n channels are identical, i.e., $\tau_i=\tau,\ 1\le i\le n,$ equalities (1.1) provide the Sevastyanov formulas [18] for the $\mathrm{M/G/n/0}$ system.

In the system with heterogeneous channels, it is of interest to calculate the utilization coefficient of each server. Let U_i denotes the utilization coefficient of the channel with service-time mean τ_i and $\overline{U}$ is the average number of busy channels. Since

$$\overline{U} = \sum_{k=1}^{n} kp_k = p_0 \sum_{i=1}^{n} \tau_i \left(\sum_{k=2}^{n} \frac{\lambda^k}{A_n^k} \sum_{\substack{i_1, i_2, \ldots, i_{k-1}=1 (\neq i); \\ i_1 < i_2 < \ldots < i_{k-1}}}^{n} \prod_{j=1}^{k-1} \tau_{i_j} + \frac{\lambda}{n} \right)$$

and, on the other hand,

$$\overline{U} = \sum_{i=1}^{n} U_i, \tag{1.6}$$

we have

$$U_i = p_0 \tau_i \left(\sum_{k=2}^{n} \frac{\lambda^k}{A_n^k} \sum_{\substack{i_1, i_2, \ldots, i_{k-1}=1 (\neq i); \\ i_1 < i_2 < \ldots < i_{k-1}}}^{n} \prod_{j=1}^{k-1} \tau_{i_j} + \frac{\lambda}{n} \right), \qquad 1 \le i \le n. \tag{1.7}$$

1.1.2 An example of calculation of the stationary characteristics

Let $n = 4$, $\lambda = 2$, $\tau_1 = 4$, $\tau_2 = 3$, $\tau_3 = 2$, and $\tau_4 = 1$. Table 1.1 contains the stationary characteristics of the $\mathrm{M_{1/n(free)}/G_1,\ldots,G_n/n/0}$ system. Their values were calculated according to the formulas (1.1) and (1.5)-(1.7). For comparison, Table 1.1 presents the values of these characteristics obtained with the help of the GPSS World simulation system, which were determined at the simulation time $T_{\mathrm{mod}} = 10^5$. Simulation results were obtained for the following service-time distributions:

(a) uniform distributions on intervals [3, 5], [2, 4], [1, 3], and [0.5, 1.5] in the first to fourth channels, respectively;

(b) exponential distributions with mean values $\tau_1 = 4$, $\tau_2 = 3$, $\tau_3 = 2$, and $\tau_4 = 1$;

(c) deterministic values $\tau_1 = 4$, $\tau_2 = 3$, $\tau_3 = 2$, and $\tau_4 = 1$;

(d) uniform distribution on the interval [3, 5], exponential distribution with the mean value $\tau_2 = 3$, the deterministic value $\tau_3 = 2$, and the uniform distribution on the interval [0.5, 1.5] in the first to fourth channels, respectively.

Obtained data confirm the insensitivity of the stationary characteristics of the $\mathrm{M_{1/n(free)}/G_1,\ldots,G_n/n/0}$ system with respect to the service-time distributions.

Table 1.1

Method, variant of the distributions	p_0	p_1	p_2	p_3	p_4	U_1	U_2	U_3	U_4	$\overline{U}$	P_{rej}
Analytical	0.020	0.099	0.232	0.331	0.318	0.808	0.765	0.695	0.560	2.828	0.318
GPSS World, (a)	0.019	0.100	0.233	0.330	0.318	0.809	0.765	0.694	0.560	2.828	0.319
GPSS World, (b)	0.021	0.995	0.230	0.332	0.318	0.806	0.765	0.696	0.559	2.828	0.319
GPSS World, (c)	0.020	0.098	0.235	0.330	0.316	0.809	0.763	0.695	0.557	2.823	0.317
GPSS World, (d)	0.020	0.100	0.232	0.333	0.316	0.809	0.761	0.696	0.559	2.825	0.318

Let us give the text of the used program of GPSS World.

```
; The model 1.1
Lam   EQU 2
Prej   VARIABLE 1-N$LT/N$L0
Tmod   EQU 100000
; Boolean variables
Ver1234   BVARIABLE F1'AND'F2'AND'F3'AND'F4
Ver123   BVARIABLE F1'AND'F2'AND'F3
Ver124   BVARIABLE F1'AND'F2'AND'F4
Ver134   BVARIABLE F1'AND'F3'AND'F4
Ver234   BVARIABLE F2'AND'F3'AND'F4
Ver12   BVARIABLE F1'AND'F2
Ver13   BVARIABLE F1'AND'F3
Ver14   BVARIABLE F1'AND'F4
Ver23   BVARIABLE F2'AND'F3
Ver24   BVARIABLE F2'AND'F4
Ver34   BVARIABLE F3'AND'F4
Ver1   BVARIABLE F1
Ver2   BVARIABLE F2
Ver3   BVARIABLE F3
Ver4   BVARIABLE F4
Dis   TABLE (F1+F2+F3+F4),0,1,7
GENERATE 1
TABULATE Dis
TERMINATE
GENERATE (Exponential(1,0,(1/Lam)))
; Distribution of customers between the channels
L0   TEST E BV$Ver1234,0,OUT
TEST E BV$Ver123,0,L4
TEST E BV$Ver124,0,L3
TEST E BV$Ver134,0,L2
```

```
TEST E BV$Ver234,0,L1
TEST E BV$Ver12,0,L5
TEST E BV$Ver13,0,L6
TEST E BV$Ver14,0,L7
TEST E BV$Ver23,0,L8
TEST E BV$Ver24,0,L9
TEST E BV$Ver34,0,L10
TEST E BV$Ver1,0,L11
TEST E BV$Ver2,0,L14
TEST E BV$Ver3,0,L17
TEST E BV$Ver4,0,L20
TRANSFER PICK,L23,L24
L23   TRANSFER ,L1
TRANSFER ,L2
TRANSFER ,L3
L24   TRANSFER ,L4
L5   TRANSFER 500,L3,L4
L6   TRANSFER 500,L2,L4
L7   TRANSFER 500,L2,L3
L8   TRANSFER 500,L1,L4
L9   TRANSFER 500,L1,L3
L10   TRANSFER 500,L1,L2
L11   TRANSFER PICK,L12,L13
L12   TRANSFER ,L2
TRANSFER ,L3
L13   TRANSFER ,L4
L14   TRANSFER PICK,L15,L16
L15   TRANSFER ,L1
TRANSFER ,L3
L16   TRANSFER ,L4
L17   TRANSFER PICK,L18,L19
L18   TRANSFER ,L1
TRANSFER ,L2
L19   TRANSFER ,L4
L20   TRANSFER PICK,L21,L22
L21   TRANSFER ,L1
TRANSFER ,L2
L22   TRANSFER ,L3
; Channel 1
L1   TRANSFER BOTH,,OUT
SEIZE 1
ADVANCE  (Uniform(1,3,5))
;ADVANCE  (Exponential(1,0,4))
```

```
;ADVANCE  4
RELEASE 1
TRANSFER ,LT
; Channel 2
L2   TRANSFER BOTH,,OUT
SEIZE 2
ADVANCE  (Uniform(1,2,4))
;ADVANCE  (Exponential(1,0,3))
;ADVANCE  3
RELEASE 2
TRANSFER ,LT
; Channel 3
L3   TRANSFER BOTH,,OUT
SEIZE 3
ADVANCE  (Uniform(1,1,3))
;ADVANCE  (Exponential(1,0,2))
;ADVANCE  2
RELEASE 3
TRANSFER ,LT
; Channel 4
L4   TRANSFER BOTH,,OUT
SEIZE 4
ADVANCE (Uniform(1,0.5,1.5))
;ADVANCE  (Exponential(1,0,1))
;ADVANCE  1
RELEASE 4
LT   TERMINATE
OUT   TERMINATE
GENERATE Tmod
SAVEVALUE Prj,V$Prej
TERMINATE 1
START 1
```

1.1.3 The $\mathbf{M^{X}_{1/n(free)}/G_1,\ldots,G_n/n/0}$ system

Let us consider the $M^{X}_{1/n(free)}/G_1,\ldots,G_n/n/0$ system with batch arrivals. The interarrival times are independent, exponentially distributed random variables with parameter λ. We introduce the notation: X is the number of customers in the batch, $E(X)$ is the mathematical expectation of the random variable X, a_k $(1\le k\le L)$ is probability of the event $\{X=k\}$, and $a_1+a_2+\ldots+a_L=1$.

Let us prove that *the stationary characteristics of the* $\mathrm{M}^{\mathrm{X}}_{1/\mathrm{n(free)}}/\mathrm{G}_1,\ldots,\mathrm{G}_\mathrm{n}/\mathrm{n}/0$ *system do not have the insensitivity property with respect to the form of the service-time distributions*.

Let $n=2$, and $L=2$. Let us calculate the stationary characteristics for two variants of the service-time distributions in the two channels of the system.

Variant A. The exponential distributions with parameters μ_1 and μ_2 respectively.

Let us enumerate the system's states as follows: s_0 corresponds to the empty system; s_1 is the state, when one channel is busy; $s_{1,0}$ is the state, when the first channel is busy, and second channel is free; $s_{0,1}$ is the state, when the first channel is free, and second channel is busy; s_2 is the state, when both channels are busy. We denote by p_i and $p_{i,j}$, stationary probabilities that the system is in the state s_i and $s_{i,j}$ respectively, then $p_1 = p_{1,0} + p_{0,1}$. To calculate the stationary probabilities, we obtain the system:

$$\begin{aligned}
&-\lambda p_0 + \mu_1 p_{1,0} + \mu_2 p_{0,1} = 0;\\
&-(\lambda+\mu_1)p_{1,0} + 0{,}5\lambda a_1 p_0 + \mu_2 p_2 = 0;\\
&-(\lambda+\mu_2)p_{0,1} + 0{,}5\lambda a_1 p_0 + \mu_1 p_2 = 0;\\
&p_0 + p_{1,0} + p_{0,1} + p_2 = 1.
\end{aligned} \tag{1.8}$$

Variant B. We have second-order Erlangian distribution with parameter $2\mu_1$ in the first channel, and exponential distribution with parameter μ_2 in the second channel.

Using the method of phases, we represent the service time in the first channel, distributed according to the second-order Erlang law, in the form of $T_1 + T_2$. Here, the random variables T_1 and T_2 are distributed exponentially with parameter $2\mu_1$.

Let us enumerate the system's states as follows: s_0 corresponds to the empty system; $s^{(i)}_{1,0}$ is the state, when the first channel is busy and service occurs in the *i*th phase ($i=1,2$), and second channel is free; $s_{0,1}$ is the state, when the first channel is free, and second channel is busy; $s^{(i)}_2$ is the state, when both channels are busy and service occurs in the *i*th phase ($i=1,2$) in the first channel. We denote by $p_0, p^{(1)}_{1,0}, p_{0,1}$, $p^{(2)}_{1,0}, p^{(1)}_2$ and $p^{(2)}_2$ respectively, stationary probabilities that the system is in the each of these states. To calculate the stationary probabilities, we obtain the system:

$$
\begin{aligned}
&-\lambda p_0 + 2\mu_1 p_{1,0}^{(2)} + \mu_2 p_{0,1} = 0;\\
&-(\lambda + \mu_2) p_{0,1} + 0,5\lambda a_1 p_0 + 2\mu_1 p_2^{(2)} = 0;\\
&-(\lambda + 2\mu_1) p_{1,0}^{(1)} + 0,5\lambda a_1 p_0 + \mu_2 p_2^{(1)} = 0;\\
&-(\lambda + 2\mu_1) p_{1,0}^{(2)} + 2\mu_1 p_{1,0}^{(1)} + \mu_2 p_2^{(2)} = 0;\\
&-(2\mu_1 + \mu_2) p_2^{(1)} + \lambda a_2 p_0 + \lambda (p_{1,0}^{(1)} + p_{0,1}) = 0;\\
&p_0 + p_{0,1} + p_{1,0}^{(1)} + p_{1,0}^{(2)} + p_2^{(1)} + p_2^{(2)} = 1.
\end{aligned} \tag{1.9}
$$

Let $\lambda = 2$, $\mu_1 = 1$, $\mu_2 = 0.1$, $a_1 = 0.25$, and $a_2 = 0.75$. In Table 1.2 we present the stationary characteristics of the $\mathrm{M}^{\mathrm{X}}_{1/2(\mathrm{free})}/\mathrm{G}_1,\mathrm{G}_2/2/0$ system calculated by using the solutions of the systems (1.8) and (1.9).

Table 1.2

Variant of the distributions	p_0	p_1	p_2	$\mathrm{E}(N_\mathrm{c})$	$\mathrm{P}_{\mathrm{rej}}$
A	0.027233	0.331998	0.640769	1.613536	0.216946
B	0.028264	0.330143	0.641593	1.613329	0.216918

Note that $\overline{U} = \mathrm{E}(N_\mathrm{c})$, since for the system with rejections the utilization coefficient is equal to the mean value N_c of the stationary number of customers in the system. Stationary value of the rejection probability is calculated as the ratio of the time-weighted numbers of lost and arrived customers. Formulas to determine $\mathrm{P}_{\mathrm{rej}}$ for the cases *A* and *B* have the form

$$
\mathrm{P}_{\mathrm{rej}} = 1 - \frac{1}{\lambda \mathrm{E}(X)} \big(\mu_1 p_{1,0} + \mu_2 p_{0,1} + (\mu_1 + \mu_2) p_2 \big); \tag{1.10}
$$

$$
\mathrm{P}_{\mathrm{rej}} = 1 - \frac{1}{\lambda \mathrm{E}(X)} \big(2\mu_1 (p_{1,0}^{(2)} + p_2^{(2)}) + \mu_2 (p_{0,1} + p_2) \big). \tag{1.11}
$$

The data of Table 1.2 show that the insensitivity property of the stationary characteristics of the $\mathrm{M}^{\mathrm{X}}_{1/n(\mathrm{free})}/\mathrm{G}_1,\ldots,\mathrm{G}_n/\mathrm{n}/0$ system with respect to the form of the service-time distributions is not saved.

Remark 1.2. *In the case when* $a_2 = 1$, *the* $\mathrm{M}^{\mathrm{X}}_{1/2(\mathrm{free})}/\mathrm{G}_1,\mathrm{G}_2/2/0$ *system has the insensitivity property of the values* $\mathrm{E}(N_\mathrm{c})$ *and* $\mathrm{P}_{\mathrm{rej}}$. *However, in this case, the equiprobable distribution of customers by free channels, it is the same as the ordered distribution of the customers in the* $\mathrm{M}^{\mathrm{X}}/\mathrm{G}_1,\mathrm{G}_2/2/0$ *system. So, we give proof of the insensitivity when considering this system.*