The main advantage of simulation over analytical modeling is universality in the sense of the possibility of studying any complex queueing systems, taking into account such factors and conditions that are difficult or impossible to consider by analytical modeling. GPSS World is the General Purpose Simulation System for creating simulation models designed for use in Windows environments. Each person is inclined to perceive information differently. Most people are easier to learn new things on concrete examples. The proposed book is constructed as a collection of programs for the GPSS World environment, containing the author's models of various queuing systems. Each model is accompanied by detailed step by step explanations. Considerable attention is paid to the creation of models of systems and certain aspects of modeling, insufficient lighting in the literature: systems with threshold strategies of functioning, with batch arrivals, with a separate queue for each channel, retrial systems, closed systems, determination of employment characteristics of separate channels and an entire system. The book is intended for researchers and students engaged in simulation of queueing systems.

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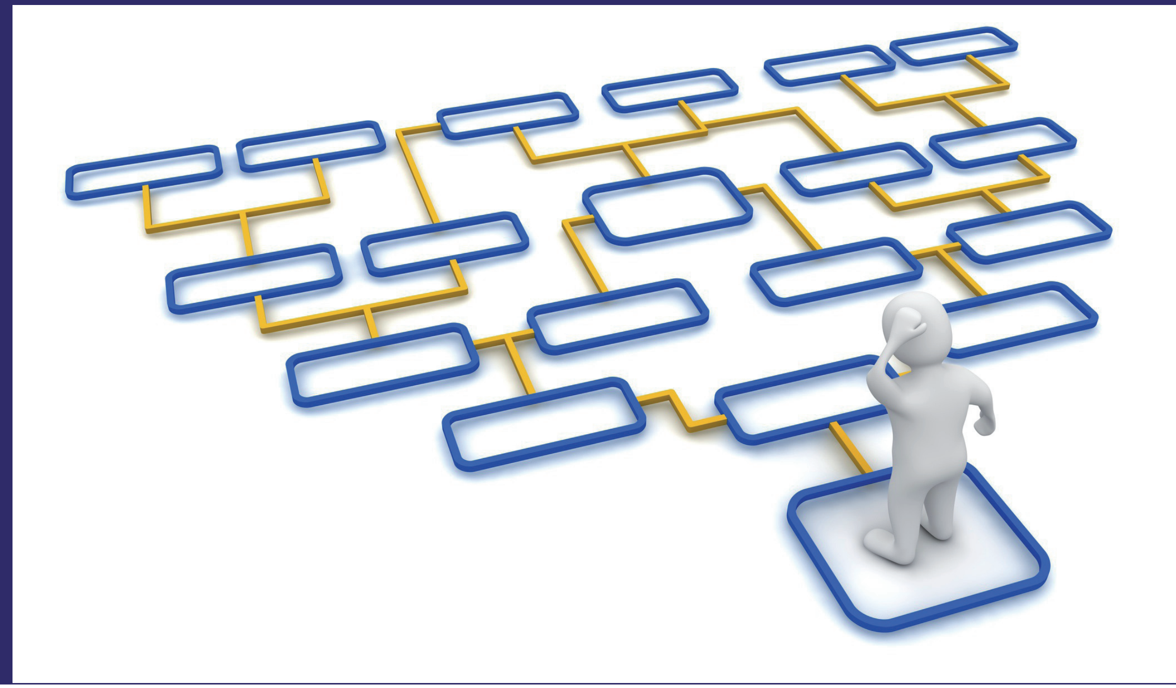
Models of queueing systems

Zhernovyi



Yuriy Zhernovyi

Creating models of queueing systems using GPSS World

Programs, detailed explanations and analysis of results

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INTRODUCTION

In 2000 the company Minuteman Software has developed a system simulation GPSS World. GPSS World is based on the seminal language of computer simulation, GPSS, which stands for General Purpose Simulation System.

This language was created by Geoffrey Gordon in 1960. The basis of simulation algorithms in GPSS is a discrete-event approach. In GPSS developers were able to very clearly and gracefully pass on the verge of the correspondence with the problem area (in the terminology, by destination, by research methodology) and the effectiveness of programming (modeling convenience, speed, use of computer resources). GPSS World is the most modern version of GPSS for Windows.

A dynamic element of a model created in the environment of GPSS World, is a transaction. A transaction is an abstract object that moves between static elements, reproducing the various events of the real object being modeled. Static elements of a model are sources of Transactions, Storages, Queues and others. Their location in the model is defined by Blocks.

Model, Simulation, and Report Objects form the 3 basic types that are used in all GPSS World simulations. Typically, a Model is developed by editing the statements in a Model Object. Then, a Create Simulation command is issued, thereby creating a Simulation Object structured according to the statements in the Model. Normally, when a simulation completes, a Report Object is created automatically.

GPSS World has a scheduler that performs the following functions:

- ensuring of movement of dynamic objects (transactions) by the routes specified by the developer;
- planning of events, occurring in the model, by recording the time of occurrence of each event and fulfillment of them in increasing time sequence;
- registration of statistical information on the operation of the model;
- promotion of model time during simulation.

The system clock, keeping the values of the absolute system clock, ensures the correct processing sequence of events in time.

There are now sufficient textbooks in which the basics of building and operation of the GPSS World system are set out [2, 3, 5, 6, 11]. Study a user manual on GPSS World with a detailed description of all blocks and their interaction takes a lot of time and is boring, if the process of learning is not backed by specific examples. Therefore, the proposed book is built as a collection of author simulation models of various queueing systems. Each model is accompanied by detailed step by step explanations. Considerable attention is paid to the creation of models of systems and certain aspects of modeling, insufficient lighting in the literature: systems with threshold strategies of functioning [7–10, 13], with batch arrivals, with a separate queue for each channel, with heterogeneous channels, retrial systems, closed systems, determination of employment characteristics of separate channels and an entire system.

Before you begin the computation of stationary characteristics of a queueing system using a simulation model, we must ensure that a random process describing the operation of the system, has a limiting stationary process. The operating mode of the system, corresponding to this process, is called a *steady-state* or *stationary*. For each studied queueing system without restrictions on the queue length, we specify conditions for the existence of a steady state. For systems with rejections and with restriction on the queue length a steady state exists for all values of the parameters of the input flow and service time.

In considering each queueing system we adhere to the following sequence: a description of the system, simulation model, explanation to the constructed model, model results and discussion.

In the explanations to simulation models, we try to avoid repetitions, stopping only on the features of the functioning of those blocks and operators which appear for the first time.

For better understanding of the material we recommend to a reader to study the basics of construction and principles of operation of the GPSS World system [2, 3, 5, 6, 11], the analytical methods of queueing theory [1, 12] and the elementary concepts of probability theory [4].

The book consists of three Sections devoted to the creation of models of systems with rejections, with waiting and with the threshold functioning strategies respectively.

Let us consider the basic assumptions and the notation used in the text.

We assume that *an input flow of customers* is the ordinary (simultaneous arrival of two or more customers is impossible) stationary flow with limited aftereffects (stationary flow of Palma) [12, Section 1.1]. Stationary flow of Palma with an exponentially distributed time intervals between events is called *simplest* or *Poisson stationary*. We also consider systems with batch arrivals.

The interarrival times T_{ar} and the service times T_{sv} assumed to be independent identically distributed random variables with finite mean values.

Let $F(x)$ be the probability distribution function of the random variable T_{ar} and $G(x)$ be the probability distribution function of the random variable T_{sv}. Further, these distributions for convenience we call F distribution and

G distribution, respectively, and their mathematical expectations we denote by E_F and E_G.

Depending on the *number of channels n*, involved in the process of customer service, we distinguish *single-channel* (n=1) and *multichannel* (n>1) queueing systems.

A simulation time T_{mod}, for which obtained results provide approximate values of stationary characteristic of a system with a given accuracy, is selected by increasing the time of modeling and comparing the results of successive large its values. If with further increase of the simulation time, the module of difference of characteristic values does not exceed the specified accuracy, we can stop on the selected value.

We use the abbreviation SNA for a *System Numerical Attribute*, NSB for a *Next Sequential Block* and note that a single-channel device simulated in GPSS World by a Facility Entity and a multichannel device simulated in GPSS World by a Storage Entity.

Introduce the notation for stationary characteristics of queuing systems:

P_{sv} = the probability of service for arrived customer;

$P_{\text{rej}}=1-P_{\text{sv}}$ = the probability of rejection;

$E(X)$ = the average value (mathematical expectation) of the random variable X;

N_{c} = the number of customers in a system;

σ_{c} = the standart deviation of the random variable N_{c};

S_k = the state of the system, in which $N_{\text{c}}=k$;

T_k = the time spent by the system in the state S_k;

$p_k=P\{N_{\text{c}}=k\}$ = the stationary probability of the state S_k, i.e. the presence of k customers in the system;

n_{oc} = the number of busy channels;

$K_u=E(n_{\text{oc}})/n$ = the system utilization factor (the probability that a channel is busy);

Q = the number of customers in the queue (the queue length);

W = the waiting time in queue (the time spent waiting in the queue before service begins);

Q_{max} = the maximum length of the queue for the simulation time;

T_{rd} = the length of a turnover cycle of the system;

T_{std} = the length of an idle period;

T_{oc} = the length of a busy period;

T_{pq} = the length of the queue presence period;

T_{inc} = the length of an incomplete load period;

T_{ful} = the length of a full load period;

σ_α = the standart deviation of the random variable T_α;

T_{us}, T_{ov} = the time spent one-time by the system in the state of usual mode or overload mode respectively;

T_{ovk} = the time spent one-time by the channel number k in the state of overload mode;

T_{stdk}, T_{ock} = the lengths of an idle period and busy period for the channel number k;

T_{ocu}, T_{oco} = the lengths of the parts of the busy period corresponding to the stay of the system in the states of usual mode and overload mode.

1 SYSTEMS WITH REJECTIONS

1.1 A multichannel system with rejections

1.1.1 Description of the system

A system, in which an arrived customer is rejected if at the arrival time all channels are busy by earlier accepted customers, is called the *queueing system with rejections* [12, Section 4.2]. There is no queue in the queueing system.

We define the stationary characteristics of the system: P_{sv} is the probability of service, $P_{rej}=1–P_{sv}$ is the probability of rejection, $E(n_{oc})$ is the average number of busy channels, p_k $(0 \le k \le n)$ is the probability of the presence of k customers in the system.

In the case of exponential distributions F and G, these characteristics can be found by the known formulas [12, Section 4.2].

Parameters of the system:

n=5;

F is an uniform distribution on the interval [1, 2];

G is an uniform distribution on the interval [7, 10].

1.1.2 The simulation model

```
; Model 1.1.1
Sys   STORAGE 5
Dis   TABLE S$Sys 0,1,6
Tm   EQU 100000
GENERATE 1
TABULATE Dis
TERMINATE
Lal   GENERATE (Uniform(1,1,2))
GATE SNF Sys,OUT
ENTER Sys
ADVANCE  (Uniform(2,7,10))
LEAVE Sys
Lsv   TERMINATE
OUT   TERMINATE
GENERATE Tm
SAVEVALUE Psv,(N$Lsv/N$Lal)
SAVEVALUE Sav,SA$Sys
TERMINATE 1
START 1
```

1.1.3 Description of the Model 1.1.1

Below each fragment of the model, which we are interested, we give its description.

Sys STORAGE 5

We define a five-channel system (the Storage Entity named Sys, which can take no more than five transactions simultaneously).

Dis TABLE S$Sys 0,1,6

We define parameters of Table named Dis, which contains the distribution of the random variable S$Sys (the current value of the contents of Storage Entity Sys, that is, the number of busy channels). The random variable can take on integer values from 0 to 5. Here 0 is the upper bound of the first interval, 1 is the length of the interval, 6 is the number of frequency slots.

Tm EQU 100000

The EQU Command defines the name Tm and assigns the value 100 000 to it (the value of the simulation time).

GENERATE 1

Through each unit time the GENERATE Block creates a transaction that serves the Table Dis.

TABULATE Dis

TERMINATE

The transaction, which enters the TABULATE block, corrects the statistics of Table Dis. When the transaction enters the TERMINATE block it is removed from the simulation.

Lal GENERATE (Uniform(1,1,2))

This block creates transactions with interarrival time for the new transaction uniformly distributed on the interval [1, 2]. The number of the Random Number Generator is equal to 1.

GATE SNF Sys,OUT

The operator SNF puts the condition that the Storage Entity Sys is not filled. In the case of fulfillment of this condition the GATE block skips the transaction to the next block. If the predetermined condition is not satisfied, the GATE block sends the transaction to the label OUT for destruction.

ENTER Sys

ADVANCE (Uniform(2,7,10))

LEAVE Sys

The combination of these three blocks provides the operation of the Storage Entity Sys as the five-channel system, in which each channel service time (time delay of transaction) is uniformly distributed in the interval [7, 10]. The number of the Random Number Generator is equal to 2.

Lsv TERMINATE
OUT TERMINATE

Each of these blocks destroys entering transactions (customers leave the system). Separation of blocks caused by the necessity to compute the service probability.

GENERATE Tm

This block creates a transaction at the time specified by the value Tm (i.e., at the simulation completion time).

SAVEVALUE Psv,(NLsv/NLal)

When a transaction enters this block (the entry time is the simulation completion time), the probability of service is assigned to the Savevalue Entity named Psv. The probability is defined as the ratio of the number of served customers (the number of transactions passed through the label Lsv) to the number of arrived customers (the number of transactions passed through the label Lal).

SAVEVALUE Sav,SA$Sys

When a transaction enters this block (the entry time is the simulation completion time), the current value of the *System Numerical Attribute* (SNA) SA as the average number of busy channels of the Storage Entity Sys is assigned to the Savevalue Entity named Sav. In considered model, we can drop this block, since the value of SA$Sys is automatically displayed in a standard report among the characteristics of the Storage Entity Sys.

TERMINATE 1
START 1

Thanks to these blocks and GENERATE Tm block we implement the simulation time specified by the user variable Tm. The value 1 of operand A of the TERMINATE block specifies the number of units for which the block reduces the value of the Termination Count (i.e. the value of the operand A of the START command) when a transaction enters this TERMINATE block. Thus, the value of the Termination Count becomes 0 and the simulation is stopped.

1.1.4 The results of the Model 1.1.1

Let us present the fragment of a standard GPSS World report, generated by the Model 1.1.1 (T_{mod}=100 000).

```
STORAGE    CAP. REM. MIN. MAX.  ENTRIES AVL.  AVE.C. UTIL. RETRY DELAY
 SYS        5    1    0    5    51060   1     4.338  0.868    0    0
```

```
TABLE     MEAN    STD.DEV.       RANGE           RETRY FREQUENCY CUM.%
 DIS      4.338    0.683                           0
                              _  -        0.000              1     0.00
                          0.000  -        1.000              2     0.00
                          1.000  -        2.000            521     0.52
                          2.000  -        3.000          10567    11.09
                          3.000  -        4.000          43461    54.55
                          4.000  -        5.000          45447   100.00

SAVEVALUE               RETRY        VALUE
 PSV                     0           0.766
 SAV                     0           4.338
```

In Table STORAGE/SYS we find the following useful information: the number of customers accepted for service equals 51060, $E(n_{oc})$=4.338 is the average number of busy channels, $K_u=E(n_{oc})/n$=0.868 is the utilization factor of the system (or the probability that a channel is busy).

Tables SAVEVALUE/PSV,SAV give the service probability value P_{sv}=0.766 (therefore, the rejection probability P_{rej}=0.234) and value $E(n_{oc})$=4.338, which we specifically allocate separately in Savevalue Entity.

With the help of TABLE/DIS we can determine the stationary distribution probabilities of the number of customers in the system: p_0=0.00001, p_1=0.00002, p_2=0.00521, p_3=0.10567, p_4=0.43461, p_5=0.45447. To do this, we divide the corresponding values of column FREQUENCY on the simulation time value. The average number of customers in the system $E(N_c)$=4.338 for the system with rejections coincides with the average number of busy channels $E(n_{oc})$.

1.1.5 The single-channel system

For a single-channel system, we construct a simulation model by making minor changes to the Model 1.1.1.

It is sufficient for this to replace

```
Sys   STORAGE 5
```

block by

```
Sys   STORAGE 1
```

The second way is associated with the peculiarities of representation of some blocks and operators which are only used for a single-channel system, and requires a larger number of changes. As a result, we obtain the following model:

```
; Model 1.1.2
Dis   TABLE F$Sys 0,1,2
Tm    EQU 100000
```

```
GENERATE 1
TABULATE Dis
TERMINATE
Lal   GENERATE (Uniform(1,1,2))
GATE NU Sys,OUT
SEIZE Sys
ADVANCE  (Uniform(2,7,10))
RELEASE Sys
Lsv   TERMINATE
OUT   TERMINATE
GENERATE Tm
SAVEVALUE Psv,(N$Lsv/N$Lal)
SAVEVALUE Fav,(FR$Sys/1000)
TERMINATE 1
START 1
```

Let us present the fragment of a standard GPSS World report, generated by the Model 1.1.2 (T_{mod}=100 000).

```
FACILITY ENTRIES UTIL.   AVE. TIME AVAIL. OWNER PEND INTER RETRY DELAY
 SYS      10787   0.916       8.489  1   166670    0    0     0      0

TABLE      MEAN    STD.DEV.       RANGE             RETRY FREQUENCY CUM.%
 DIS      0.915    0.279                              0
                                _  -        0.000          8513     8.51
                           0.000  -  _                    91487   100.00

SAVEVALUE               RETRY       VALUE
 PSV                      0          0.162
 FAV                      0          0.916
```

Here, in Table FACILITY/SYS we can find information which was not in the table for the multichannel system. The average time during which one transaction occupies the Facility Entity (a single-channel device) equals 8.489 (approximately equal to the mean for the uniform distribution on the interval [7, 10]). The number of transaction which occupies the Facility Entity at the simulation completion time equals 166670. The average number of busy channels, which for a single-channel system coincides with the utilization factor of the system, is equal to 0.916. The same value we find in Table SAVEVALUE/PSV, FAV, and here we have the service probability P_{sv}=0.162, which, of course, is much less than for the five-channel system.

Let us now consider the changes made to the Model 1.1.1.

The string

```
Dis    TABLE S$Sys 0,1,6
```

is replaced by the following:

```
Dis    TABLE F$Sys 0,1,2
```

This replacement is associated with using of others SNAs for Facilities. The SNA F takes the value 1 if a Facility Entity is busy, and value 0 if a Facility Entity is free.

For the same reason, we have made changes to

```
SAVEVALUE Sav,SA$Sys
```

block. Now in this block there is the SNA FR, which defines the utilization factor of a Facility Entity in parts-per-thousand.

Instead of a pair of blocks

```
ENTER Sys
LEAVE Sys
```

for a Facility Entity there are the following:

```
SEIZE Sys
RELEASE Sys
```

In this case, description, which indicates a name and number of a Storage channels,

```
Sys    STORAGE 5
```

is not used.

The last change is related to using

```
GATE NU Sys,OUT
```

block. Here the SNA NU means that the Facility Entity Sys is free.

1.1.6 Obtaining statistics for each channel

For the system, which is described in Section 1.1.1, we construct a model, allowing us to collect statistical information for each channel separately.

```
; Model 1.1.3
Dis   TABLE (F1+F2+F3+F4+F5) 0,1,6
Tm    EQU 100000
GENERATE 1
TABULATE Dis
TERMINATE
Lal   GENERATE (Uniform(1,1,2))
GATE NU 1,TR2
SEIZE 1
ASSIGN 1,1
TRANSFER ,ADV
TR2   GATE NU 2,TR3
SEIZE 2
ASSIGN 1,2
```

```
TRANSFER ,ADV
TR3   GATE NU 3,TR4
SEIZE 3
ASSIGN 1,3
TRANSFER ,ADV
TR4   GATE NU 4,TR5
SEIZE 4
ASSIGN 1,4
TRANSFER ,ADV
TR5   GATE NU 5,OUT
SEIZE 5
ASSIGN 1,5
ADV   ADVANCE (Uniform(2,7,10))
Lsv   RELEASE P1
OUT   TERMINATE
GENERATE Tm
SAVEVALUE Psv,(N$Lsv/N$Lal)
SAVEVALUE Fav,((FR1+FR2+FR3+FR4+FR5)/1000)
TERMINATE 1
START 1
```

Let us give a description of the Model 1.1.3, confining blocks, additionally introduced in comparison with the Model 1.1.1.

Dis   TABLE (F1+F2+F3+F4+F5) 0,1,6

The SNA F takes the value 1 if a Facility Entity is busy, and value 0 if a Facility Entity is free. As a result of summation we get the current value of the number of customers in the five-channel system.

GATE NU 1,TR2

The GATE block provides operation of the first channel in the mode of a system with rejections. The SNA NU means that the Facility Entity 1 is free. If it is free, then the transaction enters the next block. Otherwise it goes to the label TR2, where the similar test is performed for the second channel.

SEIZE 1

This block enables a transaction to acquire ownership of a Facility Entity 1 (it registers customers entered the first channel).

ASSIGN 1,1

The value 1 (a channel number) is assigned to parameter number 1 of the entering transaction.

TRANSFER ,ADV

When a transaction enters this block, it is immediately scheduled for the block at location ADV.

TR5 GATE NU 5,OUT

The GATE block provides operation of the fifth channel in the mode of a system with rejections. If the channel is busy, then the active transaction is directed to the label OUT for the destruction.

Lsv RELEASE P1

This block registers the transactions, which leave the Facility Entity after a delay for service. The SNA P1 denotes a value of the parameter number 1 of the active transaction. This parameter takes the values from 1 to 5 depending on which of the blocks from SEIZE 1 to SEIZE 5 the transaction is registered.

SAVEVALUE Fav,((FR1+FR2+FR3+FR4+FR5)/1000)

The SNA FR defines the utilization factor of a Facility Entity in parts-per-thousand. The Savevalue Entity named Fav allows us to define the average number of busy channels ($E(n_{oc})$) for the five-channel system. To compute $E(n_{oc})$ in other way is sufficient to add the values obtained in a standard report for each channel in the column UTIL.

Here is the fragment of a standard report for the Model 1.1.3:

```
FACILITY ENTRIES UTIL.  AVE. TIME AVAIL. OWNER PEND INTER RETRY DELAY
 1       10782   0.916       8.498  1        0    0    0     0      0
 2       10561   0.898       8.504  1   166681    0    0     0      0
 3       10315   0.876       8.492  1   166684    0    0     0      0
 4        9950   0.845       8.488  1   166688    0    0     0      0
 5        9452   0.803       8.493  1   166686    0    0     0      0

TABLE      MEAN    STD.DEV.       RANGE           RETRY FREQUENCY CUM.%
 DIS       4.338    0.683                             0
                                _  -       0.000                1     0.00
                           0.000   -       1.000                2     0.00
                           1.000   -       2.000              521     0.52
                           2.000   -       3.000            10567    11.09
                           3.000   -       4.000            43461    54.55
                           4.000   -  _                     45447   100.00

SAVEVALUE               RETRY        VALUE
 PSV                      0           0.766
 FAV                      0           4.338
```

Statistical data given in Tables TABLE/DIS and SAVEVALUE/PSV,FAV, coincide with the data given in Section 1.1.2. In Table FACILITY we find the data for each channel separately. Here ENTRIES is the number of customers entered each channel, UTIL. is the utilization factor of each channel (the sum of these characteristics gives the value of the average number of busy channels, presented in Table SAVEVALUE/FAV), AVE.TIME is the average service time of one customer for each channel, OWNER is a list of numbers of transactions which occupy the Facilities at the simulation completion time.