The purpose of the monograph is to analyze the peculiarities of applying hyperexponential distributions of order k (k=2,...,6) with both real and paradoxical and complex-valued parameters for the approximate calculation of steady-state distributions of the number of customers in non-Markovian queueing systems by the fictitious phase method. We considered not only open, but also closed queueing systems, as well as single-channel systems with a threshold change of service times and with changes of service times depending on the number of customers in the system. We check the accuracy of the obtained approximate distributions of the number of customers in the queueing system using the potentials method (for single-channel systems) and the GPSS World simulation system. For systems with the gamma distributions and Weibull distributions, the conditions on the values of the variation coefficients are indicated, for which the best accuracy of calculating the steady-state probabilities is achieved compared with the results of simulation modeling. The book is intended for researchers and students engaged in simulation of queueing systems.

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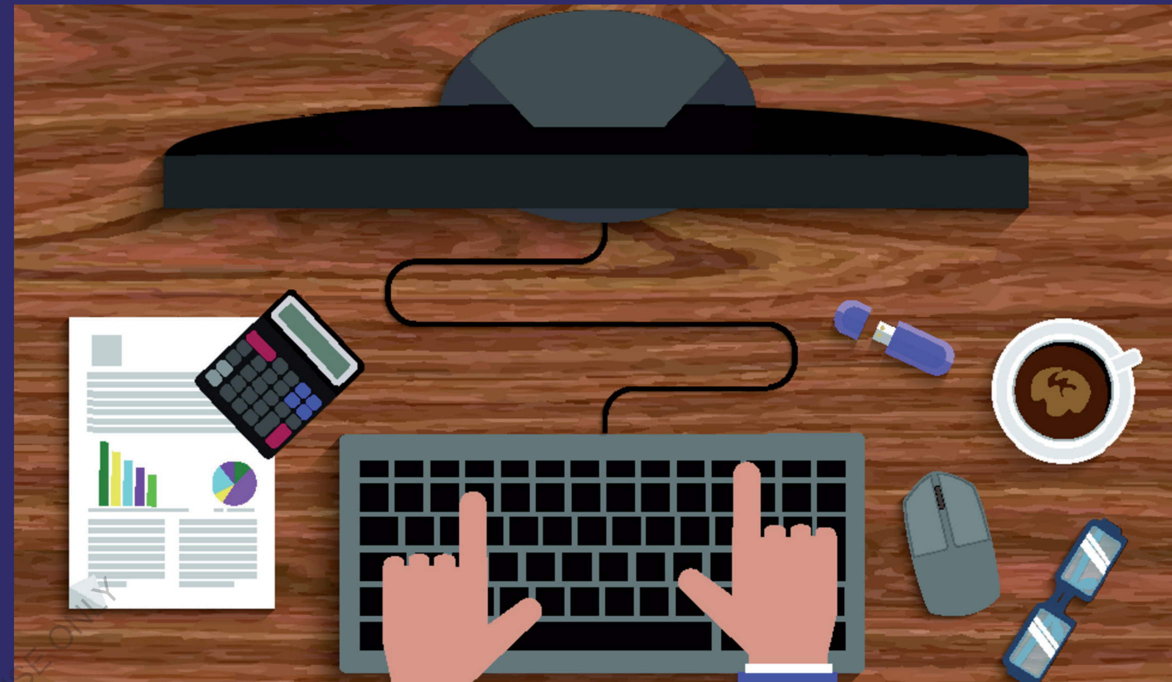
Computing non-Markovian queues

Zhernovyi



Yuriy Zhernovyi

Computing non-Markovian queues using hyperexponential distributions

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INTRODUCTION

For the study of the non-Markovian process in queueing systems, phase-type distributions are used with exponential distributions of delays in the phases. In the case of fixing the number of the phase, the states of the system has a Markov property that makes it possible to represent the transitions between them in the form of a discrete Markov process with continuous time. The idea of the fictitious phase method was proposed by A. K. Erlang. The order of approximation is the number of retained initial moments of the original distribution.

The most general form of representation of phase distributions is the Neuts' scheme [9], in which the duration of each process realization corresponds to a random time of wandering in a network with an exponential delay in each node and one absorbing state. In the case of using the Neuts' scheme, the steady-state characteristics of a queueing system are calculated in terms of Kronecker matrix operations whose numerical implementation is extremely inefficient [12]. Therefore, to approximate distributions with a coefficient of variation $V > 1$, the hyperexponential approximation (H_k) is usually used, and, in all other cases, the Erlang approximation is used. The approximation parameters are considered to be real.

Recently, interest in hyperexponential distribution has increased since its use showed its high performance in solving problems of summation of recurrent flows [11], in computing characteristics of queuing systems with "impatient" customers [10] and Jackson's networks of queueing [14], and also in analyzing stock management systems [8].

The results of applying the hyperexponential distribution of the second order (H_2) to calculating the steady-state distribution of the number of customers in the $M/G/1/\infty$ queueing system by the fictitious phase method are given in [12]. The authors allow the use of H_2-distributions with parameters of paradoxical or complex types, which makes it possible to approximate the service time with an arbitrary (in particular, smaller than one) variation coefficient. This article notes that, when calculating queueing systems with the use of H_2-approximation in the field of complex values of its parameters, a potential pathology manifests itself only in intermediate results such as the probabilities of "fictitious" microstates of transition diagram into which the "physical" system's states are split. At the stage of summation of probabilities of microstates, their complex parts are annihilated, and the components of the result of calculation, i.e., the probabilities of the number of customers in the system, become real. However, before writing this work the peculiarities of approximation using the H_k-distribution of an order $k > 2$ with paradoxical and complex parameters for calculating the *G*/*G*/*n*/*m* systems with the number of channels $n \geq 1$ remained uninvestigated.

The complex or paradoxical type of parameters of H_k-distributions emphasizes the fictitious nature of splitting a process into phases. The admissibility of complex parameters for investigating random processes was first noted by D. Cox in 1955 [4]. In [2], the authors tried to give a probabilistic interpretation of complex intensities of transitions between Markov chain states.

The purpose of the monograph is to analyze the peculiarities of applying H_k-distributions of order k $(k = 2,...,6)$ with both real and paradoxical and complex-valued parameters for the approximate calculation of steady-state distributions of the number of customers in non-Markovian queueing systems by the fictitious phase method. We considered not only open, but also closed queueing systems, as well as single-channel systems with a threshold change of service times and with changes of service times depending on the number of customers in the system.

Let us list the main stages of calculating non-Markovian queueing systems by the fictitious phase method using the hyperexponential approximation:

– the calculation of the initial moments of the distribution of service times and time intervals between customers for incoming flow;

– the calculation of parameters of H_k-distributions as solutions of the equations of the moments method;

– the compilation of the transition diagram and systems of equations for steady-state probabilities of microstates (states of a queueing system with H_k-distributions);

– the calculation and summation of probabilities of microstates by tiers and obtaining the steady-state distribution of the number of customers in the system.

In writing this book we tried to highlight the following questions:

– the study of properties of solutions of the equations of the moments method;

– the solving of the equations of the moments method for basic distributions;

– the study of properties of the "function" (pseudofunction) of hyperexponential distribution in the case of paradoxical or complex values of the distribution parameters and the influence of the value of its deviation from the true distribution function on the accuracy of the obtained approximate distribution of the number of customers;

– the analysis of the influence of increase in the number of channels of the *G*/*G*/*n*/*m* system on the accuracy of calculating the steady-state distribution of the number of customers;

– the check of the accuracy of the obtained approximate distributions of the number of customers using the potentials method [3, 17] (for single-channel systems) and the GPSS World simulation system [1, 7, 16, 18];

– the indication of ways to assess the accuracy of the obtained approximate distributions of the number of customers without the need for using simulation models;

– the determination of conditions for the variation coefficients of the gamma distributions and Weibull distributions, for which the best accuracy of calculating the steady-state distributions of the number of customers in the system is achieved compared with the results of simulation modeling;

– the analysis of the influence of load factor and queue length limiter on the accuracy of calculating steady-state probabilities.